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Using loop transformations for precision tuning in iterative programs

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Context and achievement

Context

- ⊕ Various floating-point formats exist = different level of accuracy
  - IEEE 754-2019 defines four formats: binary{16, 32, 64, 128}
  - non IEEE formats: BFloat16, Posit, ...
- ⊖ Floating-point arithmetic is non-intuitive
  - discrete and finite set of values → 0.1 not exactly representable
  - loss of arithmetic properties → $a+(b+c) \neq (a+b)+c$
- Over-sizing of the computation means → higher precision by default
- Precision tuning: technique to improve performance of numerical applications

⚠ Most existing tools do not consider iterative nature of programs ⚠

Achievement : a dynamic auto-tuning tool, targeting iterative routines

- reduce the precision of certain instructions at the iteration level,
- to the detriment of an increase of the time of tuning process.

Motivating example (1/2)

- Objective : calculate the sum $\sum_{i=1}^{1000} 0.01 = 10$

```
double s_b64 = 0.;

for (int i=1; i<=1000; i++)
  s_b64 = s_b64 + 0.01;

printf("s_b64 = %.20lf", s_b64);
```

```
s_b64 = 9.99999999999983124610
```

Motivating example (1/2)

- Objective : calculate the sum $\sum_{i=1}^{1000} 0.01 = 10$

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double s_b64 = 0.;

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```c
float s_b32 = 0.f;

for (int i=1; i<=1000; i++)
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printf("s_b32 = %.20f", s_b32);
```

```
s_b64 = 9.99999999999983124610
```

```
s_b32 = 10.00013351440429687500
```

$$\frac{|\texttt{s\_b64}-\texttt{s\_b32}|}{\texttt{s\_b64}} \approx 10^{-5}$$

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s_b64 = 9.99999999999983124610
```

```
s_b32 = 10.00013351440429687500
```

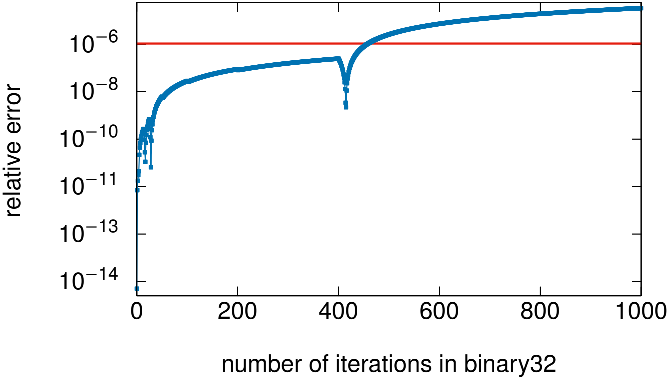
$$\frac{|\mathtt{s\_b64} - \mathtt{s\_b32}|}{\mathtt{s\_b64}} \approx 10^{-5}$$

- Only 1 instruction (addition): which format per iteration to have

$$\frac{|\mathtt{s\_b64} - \mathtt{s\_optim}|}{\mathtt{s\_b64}} < 10^{-6}?$$

☹ Existing approaches → no solution

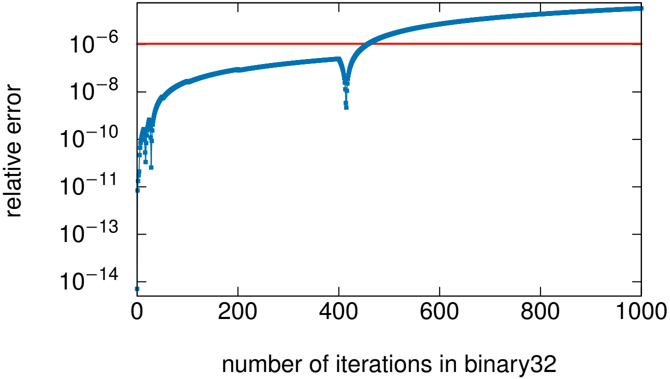
Motivating example (2/2)



- Target threshold = 10^{-6} → 458 iterations in binary32

☺ Our approach → 450 iterations in binary32

Motivating example (2/2)



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How to isolate these iterations?

Outline of the talk

1. Auto-tuning of iterative routines

2. Experimental results

3. Conclusion and perspectives

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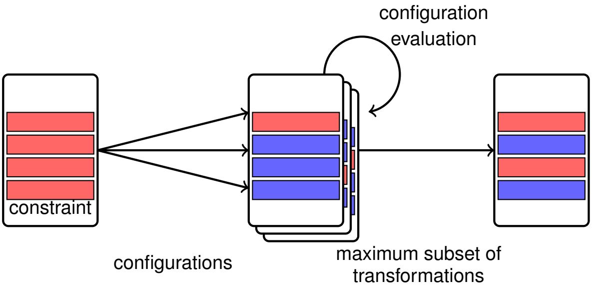
Main flow of dynamic tools

- Most dynamic tools use a trial-and-error strategy
    1. explore a set of possible transformations (configurations)
    2. evaluate the impact of each transformation (eg. accuracy)



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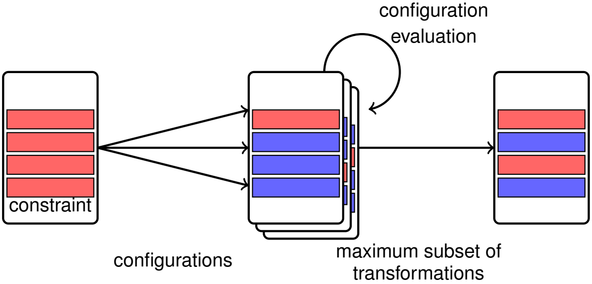


How to adapt this process to the tuning of iterative programs?

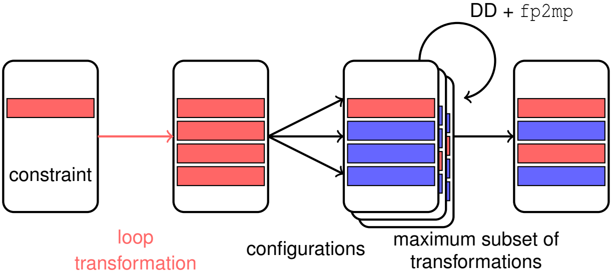
Outline of our project

Originality of the proposed approach

- change combinatorics by targeting instructions in loop bodies
- use compilation techniques on loop: loop splitting and unrolling

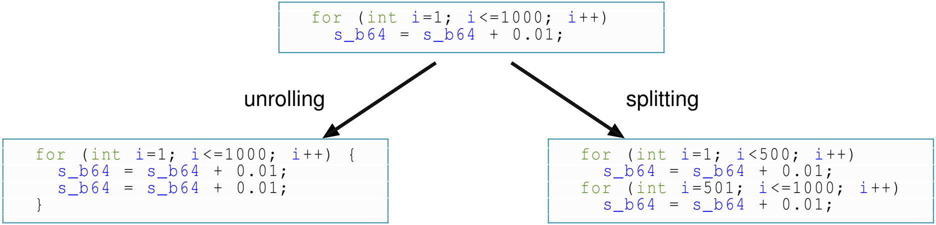
Main steps

- loop transformation (splitting, unrolling)
- configuration evaluation → fp2mp
- building of maximum subset of transformations → delta-debugging



Static loop transformation

- Objective: increase the number of possible transformations
  - leverage the LLVM capabilities of transforming programs



```
for (int i=1; i<=1000; i++)
  s_b64 = s_b64 + 0.01;
```

unrolling

```
for (int i=1; i<=1000; i++) {
  s_b64 = s_b64 + 0.01;
  s_b64 = s_b64 + 0.01;
}
```

splitting

```
for (int i=1; i<500; i++)
  s_b64 = s_b64 + 0.01;
for (int i=501; i<=1000; i++)
  s_b64 = s_b64 + 0.01;
```

  - do not modify the semantics of the program
  - allow to detect two different patterns of transformations

Static loop transformation

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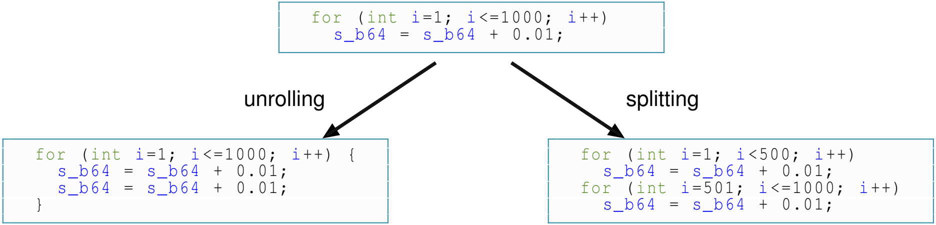

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for (int i=1; i<=1000; i++)
  s_b64 = s_b64 + 0.01;
```

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```
for (int i=1; i<=1000; i++) {
  s_b64 = s_b64 + 0.01;
  s_b64 = s_b64 + 0.01;
}
```

splitting

```
for (int i=1; i<500; i++)
  s_b64 = s_b64 + 0.01;
for (int i=501; i<=1000; i++)
  s_b64 = s_b64 + 0.01;
```

  - do not modify the semantics of the program
  - allow to detect two different patterns of transformations

⚠ Approach antagonistic to existing ones

  - current trend: reduce the combinatorics to speedup the process
  - our approach: increase the combinatorics → ☹ increase the tuning process time
    ☺ improve the quality of the tuning

Evaluate the impact of transformations

- Objective: check if the constraint is still satisfied
- Rely on fp2mp: LLVM instrumentation tool
  - duplicate floating-point instructions into their MPFR equivalent instructions
  - and allow to compute the result using a desired precision

```
double s_b64 = 0.;

for (int i=1; i<=1000; i++)
  s_b64 = s_b64 + 0.01;
printf("s_b64 = %.20lf", s_b64);

// |s_b64 - s_mpfr|/|s_b64| < 1e-6 ?
check_reverse_rel_error(s_b64, 1e-6);
```

```
double s_b64 = 0.;
// ...
mpfr_t s_mpfr, C, S;
mpfr_init2(s_mpfr, 53);
mpfr_init2(C, 53);
mpfr_init2(S, 53);
mpfr_set_d(C, 0.01, MPFR_RNDN);

for (int i=1; i<=1000; i++) {
  s_b64 = s_b64 + 0.01;
  // ...
  mpfr_set(S, s_mpfr, MPFR_RNDN);
  mpfr_add(s_mpfr, S, C, MPFR_RNDN);
}
printf("s_b64 = %.20lf", s_b64);

// |s_b64 - s_mpfr|/|s_b64| < 1e-6 ?
check_reverse_rel_error(s_b64, s_mpfr,
                                  1e-6);
mpfr_clears(s_mpfr, C, S, NULL);
```

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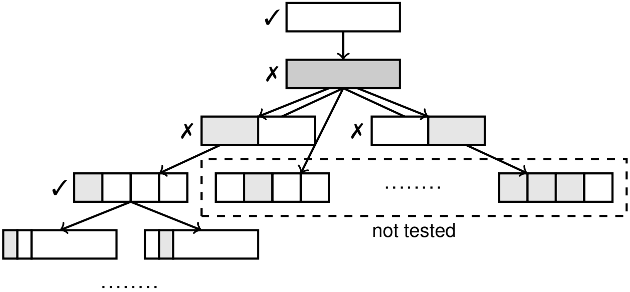
for (int i=1; i<=1000; i++) {
  s_b64 = s_b64 + 0.01;
  // ...
  mpfr_set(S, s_mpfr, MPFR_RNDN);
  mpfr_add(s_mpfr, S, C, MPFR_RNDN);
}
printf("s_b64 = %.20lf", s_b64);

// |s_b64 - s_mpfr|/|s_b64| < 1e-6 ?
check_reverse_rel_error(s_b64, s_mpfr,
                                   1e-6);
mpfr_clears(s_mpfr, C, S, NULL);
```

- Interest
  1. Apply transformations = modify MPFR initialisation precision
  2. Provide means to estimate errors due to transformations

Delta-Debugging algorithm

- Objective: isolate most relevant transformations
  - widely used in auto-tuning tools
  - `ddmax`: find a locally maximal set of changes → the contraint remains satisfied



- For each instruction → a list of possible precision (e.g. [`b32`, `b16`])
  - apply delta-debugging several times
  - find the lowest precision for each instruction

Back to motivating example

```
double s_b64 = 0.;

for (int i=1; i<=1000; i++)
  s_b64 = s_b64 + 0.01;

check_reverse_rel_error(s_b64, 1e-6);
```

- Example of our approach:
  - split iteration space $[1,1000]$ into 20 subspaces of 50 iterations

```
double s_optim = 0.;

for (int i=1; i<=50; i++)
  s_optim = s_optim + 0.01;

for (int i=51; i<=100; i++)
  s_optim = s_optim + 0.01;

for (int i=101; i<=150; i++)
  s_optim = s_optim + 0.01;

// ...
for (int i=951; i<=1000; i++)
  s_optim = s_optim + 0.01;

check_reverse_rel_error(s_optim, 1e-6);
```

- Now 20 instructions instead of 1 → 20 possible transformations
  - $2^{20} = 1048576$ possible configurations
- Finally which format per iteration to have

$$\frac{|\mathtt{s\_b64} - \mathtt{s\_optim}|}{\mathtt{s\_b64}} < 10^{-6}?$$

☺ Our tool's output: 9 instructions out of these 20 → 45% in binary32

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Impact of different strategies

	threshold	delta-debugging			split / factor = 10			split / factor = 50		
		b64 / b32	%	h:m:s	b64 / b32	%	h:m:s	b64 / b32	%	h:m:s
sum	1e-5	1 / 0	0.0	0:00:01	5 / 5	50.0	0:00:06	22 / 28	56.0	0:00:43
	1e-6	1 / 0	0.0	0:00:01	6 / 4	40.0	0:00:07	27 / 23	46.0	0:00:48
	1e-8	1 / 0	0.0	0:00:01	10 / 0	0.0	0:00:09	48 / 2	4.0	0:01:08
riemann	1e-8	3 / 2	40.0	0:00:04	15 / 35	70.0	0:00:41	10 / 240	96.0	0:01:44
	1e-9	3 / 2	40.0	0:00:04	22 / 28	56.0	0:00:53	14 / 236	94.4	0:02:04
	1e-11	5 / 0	0.0	0:00:04	44 / 6	12.0	0:01:05	19 / 231	92.4	0:02:31
arclength	1e-8	6 / 5	45.5	0:00:10	31 / 79	71.8	0:02:32	71 / 479	87.1	0:19:05
	1e-9	8 / 3	27.3	0:00:10	64 / 46	41.8	0:02:48	131 / 419	76.2	0:24:57
	1e-11	11 / 0	0.0	0:00:09	105 / 5	4.5	0:02:28	491 / 59	10.7	0:48:38
simpson	1e-7	4 / 6	60.0	0:00:06	18 / 82	82.0	0:01:42	14 / 486	97.2	0:02:13
	1e-9	6 / 4	40.0	0:00:09	58 / 42	42.0	0:02:28	182 / 318	63.6	0:27:20
	1e-11	10 / 0	0.0	0:00:09	91 / 9	9.0	0:03:21	439 / 61	12.2	0:41:46
nbody	1e-7	9 / 15	62.5	0:00:19	7 / 233	97.1	0:03:05	2 / 1198	99.8	0:02:16
	1e-10	21 / 3	12.5	0:00:27	212 / 28	11.7	0:08:17	742 / 458	38.2	5:12:40
	1e-11	24 / 0	0.0	0:00:24	228 / 12	5.0	0:07:26	1062 / 138	11.5	2:34:49
	1e-12	24 / 0	0.0	0:00:25	237 / 3	1.2	0:08:09	1160 / 40	3.3	3:03:27

Auto-tuning results for splitting strategy.

Impact of different strategies

	threshold	delta-debugging			unroll / factor = 10			unroll / factor = 50		
		b64 / b32	%	h:m:s	b64 / b32	%	h:m:s	b64 / b32	%	h:m:s
sum	1e-5	1 / 0	0.0	0:00:01	6 / 4	40.0	0:00:08	25 / 25	50.0	0:00:40
	1e-6	1 / 0	0.0	0:00:01	10 / 0	0.0	0:00:08	45 / 5	10.0	0:00:54
	1e-8	1 / 0	0.0	0:00:01	10 / 0	0.0	0:00:08	48 / 2	4.0	0:00:54
riemann	1e-8	3 / 2	40.0	0:00:04	19 / 31	62.0	0:00:29	2 / 248	99.2	0:02:00
	1e-9	3 / 2	40.0	0:00:04	25 / 25	50.0	0:00:46	43 / 207	82.8	0:04:39
	1e-11	5 / 0	0.0	0:00:04	44 / 6	12.0	0:01:02	213 / 37	14.8	0:07:36
arclength	1e-8	6 / 5	45.5	0:00:10	32 / 78	70.9	0:01:40	67 / 483	87.8	0:12:50
	1e-9	8 / 3	27.3	0:00:10	31 / 79	71.8	0:02:39	80 / 470	85.5	0:22:54
	1e-11	11 / 0	0.0	0:00:09	110 / 0	0.0	0:01:57	509 / 41	7.5	0:27:58
simpson	1e-7	4 / 6	60.0	0:00:06	10 / 90	90.0	0:00:39	23 / 477	95.4	0:03:09
	1e-9	6 / 4	40.0	0:00:09	56 / 44	44.0	0:01:24	179 / 321	64.2	0:18:18
	1e-11	10 / 0	0.0	0:00:09	95 / 5	5.0	0:02:23	413 / 87	17.4	0:39:05
nbody	1e-7	9 / 15	62.5	0:00:19	36 / 204	85.0	0:02:33	4 / 1196	99.7	0:07:55
	1e-10	21 / 3	12.5	0:00:27	206 / 34	14.2	0:09:28	900 / 300	25.0	2:33:01
	1e-11	24 / 0	0.0	0:00:24	228 / 12	5.0	0:08:29	1046 / 154	12.8	3:51:45
	1e-12	24 / 0	0.0	0:00:25	238 / 2	0.8	0:06:16	1176 / 24	2.0	1:35:16

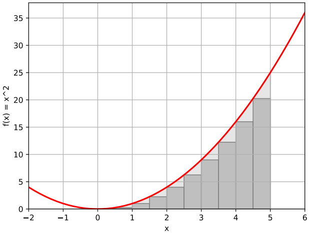
Auto-tuning results for unrolling strategy.

Focus on Riemann integral (1/3)

- Objective: compute the following integral for $f(x) = x^2$

$$\int_a^b f(x) \cdot dx \approx \frac{b-a}{n} \cdot \sum_{k=0}^{n-1} f\left(a + k \cdot \frac{b-a}{n}\right)$$

  - $[a,b] = [0,5]$
  - $n = 400$



- For performance purpose → same format for the whole loop body

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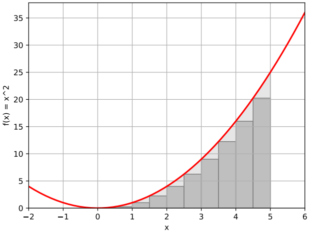
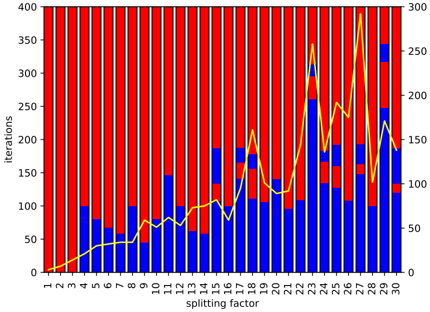


- For performance purpose → same format for the whole loop body

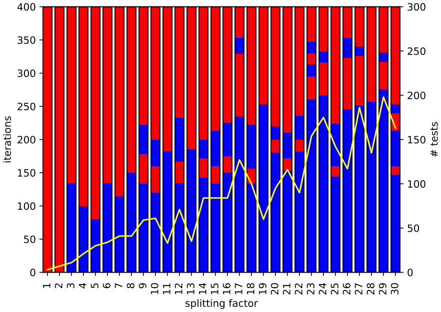
Which iterations can be done in lower precision such that the result remains at a given threshold of the binary64 result?

Focus on Riemann integral (2/3)

- Number of splittings = 1 → 30
- Available formats: binary64, binary32



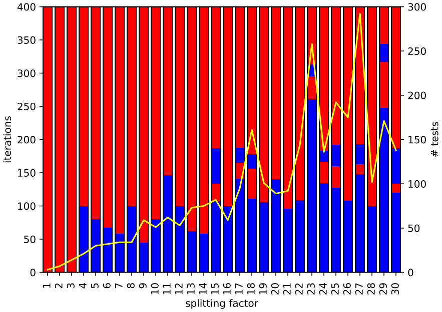
threshold = 10^{-9}



threshold = 10^{-8}

Focus on Riemann integral (3/3)

- Number of splittings = 1 → 30
- Available formats: binary64, binary32
- Only one precision change



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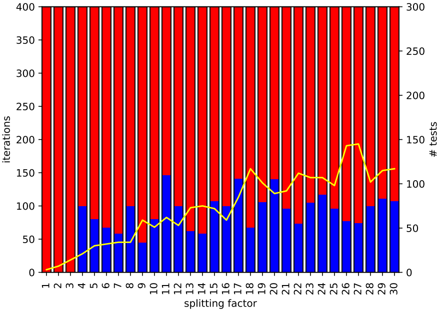

threshold = 10^{-9}, max change = 1

Auto-tuning for unbounded loops (1/2)

- Conjugate Gradient: method to solve the linear system $Ax = b$‘

```
$r_0 := p_0 := b - Ax_0$, and $k = 0$
while $\|r_k\| \geq \epsilon$ and $k <$ maxiter do
    $\alpha_k := \frac{r_k^T r_k}{p_k^T A p_k}$
    $x_{k+1} := x_k + \alpha_k p_k$
    $r_{k+1} := r_k - \alpha_k A p_k$
    $\beta_k := \frac{r_{k+1}^T r_{k+1}}{r_k^T r_k}$
    $p_{k+1} := r_{k+1} + \beta_k p_k$
    $k = k + 1$
end while
```

- In exact arithmetic, it converges in *n* iterations
- But in floating-point arithmetic, the number of iterations is linked to the precision of the computations
- Example: 494_bus matrix (Suite Sparse Matrix Collection)
    - $\epsilon = 10^{-6}$

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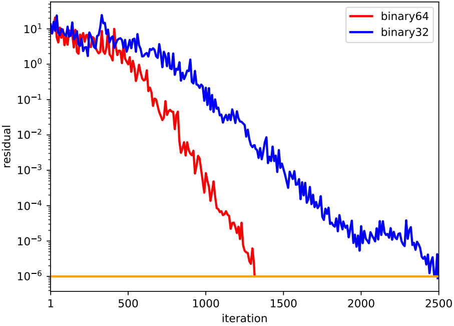

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  - binary32 = 2494 iterations

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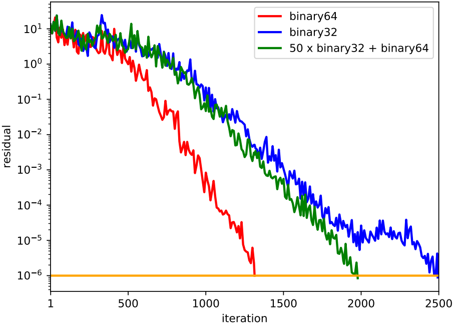

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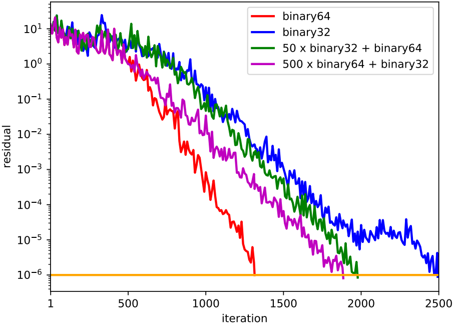

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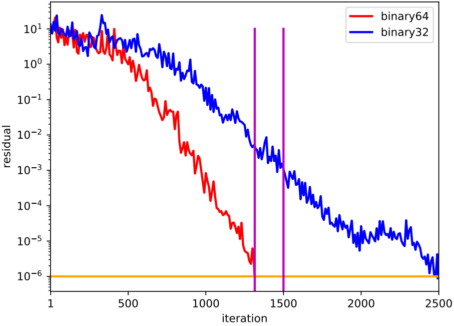

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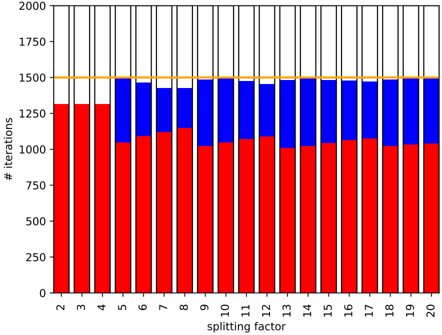
  - $\epsilon = 10^{-6}$
  - binary64 = 1315 iterations
  - binary32 = 2494 iterations

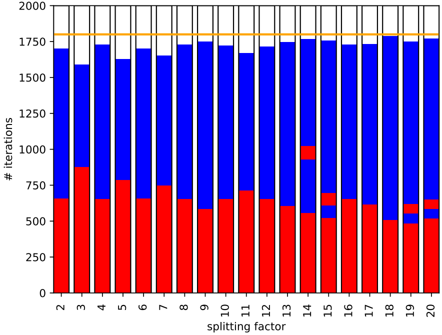
Among these 1315 iterations, which ones can be transformed into binary32, at the risk of increasing the total number of iterations?

Auto-tuning for unbounded loops (2/2)

- Number of splittings = 2 → 20
- Available formats: binary64, binary32



max iteration = 1500



max iteration = 1800

Outline of the talk

1. Auto-tuning of iterative routines

2. Experimental results

3. Conclusion and perspectives

Conclusion and perspectives

Contribution

- Dynamic tool to tune the precision of certain instructions in iterative routines
    - target instructions of loop bodies
    - based on loop transformation + fp2mp + delta-debugging

Future works

- Evaluate the speedup delivered by our tool
- Investigate other loop transformations
- Study how this approach scales → loop size, nested loops