Improving Residue-Level Sparsity in RNS-based Neural Network Hardware Accelerators via Regularization

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Overview

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RNS basics

An RNS maps an integer x to a tuple X of N residues

$$x \to X = (x_1, x_2, \ldots, x_N), \tag{1}$$

where $x_i = x \bmod m_i$ and m_i, $i = 1, 2, \ldots, N$, form a set called base $\mathcal{B}$,

$$\mathcal{B} = \{m_1, m_2, \ldots, m_N\}. \tag{2}$$

Moduli m_i of $\mathcal{B}$ are relatively co-prime; i.e.,

$$\gcd_{i \neq j}(m_i, m_j) = 1 \tag{3}$$

for all i, j, $1 \leq i, j \leq N$. The dynamic range of the representation is determined by $\mathcal{B}$, as

$$M = \prod_{i=1}^{N} m_i. \tag{4}$$

Neural Network Regularization

- Regularization is a set of strategies used in Machine Learning to reduce the generalization error, *i.e.*, overfitting.
- Modify the loss function: add regularization terms:

$$J'(\theta; X, y) = J(\theta; X, y) + a \cdot \boxed{R(\theta)} \tag{5}$$

- L1 regularization: $R(\theta) = \sum_i |\theta_i|$
- L2 regularization: $R(\theta) = \sum_i \theta_i^2$
- L1 and L2 regularization penalize large weights.

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Proposed RNS-conscious regularization

- Assume a subset $\mathcal{B}_{\text{weight}} \subset \mathcal{B}$.
- $\mathcal{B}_{\text{weight}}$ suffices to provide the dynamic range required for the representation of neural-network weights.
- The dynamic range provided by $\mathcal{B}_{\text{weight}}$ for the representation of the weights w, is

$$M_{\text{weight}} = \prod_{\forall m \in \mathcal{B}_{\text{weight}}} m. \tag{6}$$

- We propose the regularization function

$$R(w; \mathcal{B}, \mathcal{B}_w) = \lambda \prod_{i=1}^{N} \prod_{k=-K_i}^{K_i} \sigma_i \cdot (w \cdot M_{\text{weight}} - k \cdot m_i)^2, \tag{7}$$

where λ, σ_i are chosen hyperparameters and $K_i = \left\lfloor \frac{\frac{1}{2} M_{\text{weight}}}{m_i} \right\rfloor$.

Proposed RNS-conscious regularization

- The values of σ_i are chosen such that the product of (7) neither decays nor explodes.
- During training, the regularization term (7) drives a weight w, to assume a value which when converted to an integer $\widehat{w}$, is an integral multiple of m_i; therefore, the corresponding residue $\widehat{w}_i$ is zero, i.e.,

$$\widehat{w}_i = Q(wM_{\text{weight}}) \bmod m_i = 0,$$

  where $Q(x)$ rounds its argument to the nearest integer.
- In this way, the proposed regularization term increases the residue sparsity.

For the case of a network with N_w weights, the regularization term (7) can be extended as

$$R(w; \mathcal{B}, \mathcal{B}_w) = \lambda \sum_{n=1}^{N_w} \prod_{i=1}^{N} \prod_{k=-K_i}^{K_i} \sigma_i \cdot (w_n \cdot M_{\text{weight}} - k \cdot m_i)^2, \tag{8}$$

Training Results

- Test case: CNN on CIFAR-10 benchmark.

Table: CNN architecture

Layer	**Kernel Dimensions**
Convolutional 2D	$3 \times 3 \times 3 \times 32$
Max Pooling (2×2)	–
Convolutional 2D	$3 \times 3 \times 32 \times 64$
Max Pooling (2×2)	–
Convolutional 2D	$3 \times 3 \times 64 \times 64$
Fully Connected	1024×64
Fully Connected	64×10

- Assuming an RNS:

$$\mathcal{B} = \{5, 7, 31, 32, 33\}$$
$$M = 5 \cdot 7 \cdot 31 \cdot 32 \cdot 33$$

CIFAR-10 CNN Example

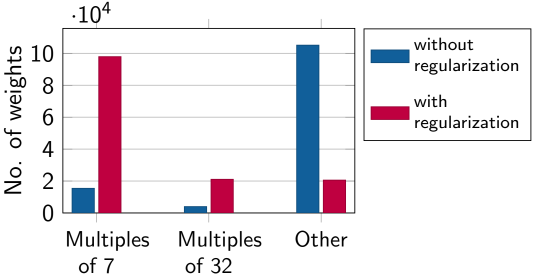
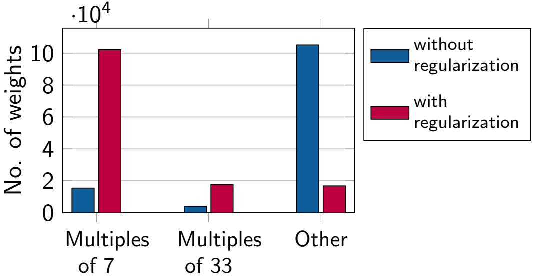


Figure: Histogram of $\lfloor w \cdot M_{\text{weight}} \rceil$ before (blue) and after (red) regularization. Base $\mathcal{B}_{\text{weight}} = \{7, 32\}$ is assumed.



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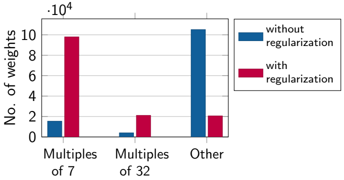

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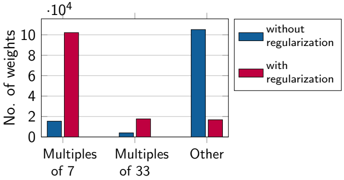

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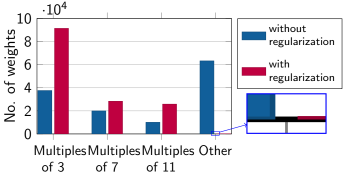


Figure: Histogram of $\lfloor w \cdot M_{\text{weight}} \rfloor$ before (blue) and after (red) regularization. Base $\mathcal{B}_{\text{weight}} = \{3, 7, 11\}$ is assumed.

VGG-16 Example

- Test case: VGG-16 on ImageNet benchmark.
- Last Fully-Connected layer (4096×1000 weights).

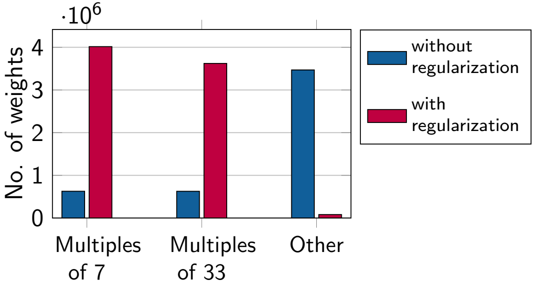


Figure: Histogram of $\lfloor w \cdot M_{\text{weight}} \rfloor$ before (blue) and after (red) regularization, for the last FC layer of VGG16 on ImageNet. Base $\mathcal{B}_{\text{weight}} = \{7, 33\}$ is assumed.

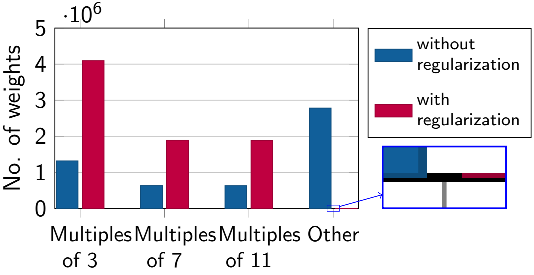


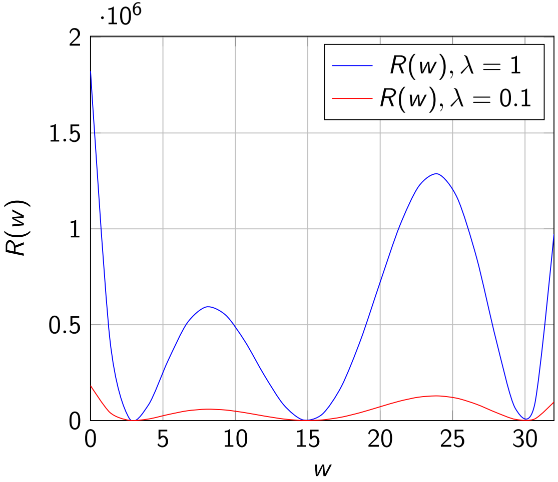
Figure: Histogram of $\lfloor w \cdot M_{\text{weight}} \rfloor$ before (blue) and after (red) regularization, for the last FC layer of VGG16 on ImageNet. Base $\mathcal{B}_{\text{weight}} = \{3, 7, 11\}$ is assumed.

Limiting the regularization loss

- Regularization loss (8) can assume extremely large values.
- Assume the regularization function:

$$R(w) = \lambda(w-3)^2(w-15)^2(w-30)^2.$$

- Parameters λ, σ_i of (8) are tuned to limit $R(w)$.
- In our experiments, $\lambda \in \{10^{-2}, 10^{-1}, 1\}$ and $\sigma_i \in [10^{-4}, 10^{-3}]$ depending on the employed RNS base.



Computational complexity

- Application of regularization loss (8) can deteriorate training speed significantly.
- For a large number N of moduli and for large K_i.
- The double product of (8) is composed of P terms,

$$P = \sum_{i=1}^{N} (2K_i + 1). \tag{9}$$

- For $\mathcal{B}_{\text{weight}} = \{7, 32\}$, $N = 2$, and $M_{\text{weight}} = 7 \cdot 32 = 224$. $K_1 = 16$, $K_2 = 3$, leading to $P = 40$ terms <u>per weight</u>.

Table: Training slowdown

$\mathcal{B}_{\textbf{weight}}$	**Slowdown**
Without regularization	$1\times$
$\{7, 32\}$	$9\times$
$\{7, 33\}$	$8.4\times$
$\{7, 9, 17\}$	$15.8\times$
$\{5, 7, 17\}$	$18.6\times$

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Hardware architectures exploiting residue sparsity

Exploit residue-level sparsity to:

- Reduce memory cost
- Reduce power consumption

Utilize low-cost moduli for the RNS bases

- $\mathcal{B} = \{5, 7, 31, 32, 33\}$
- $\mathcal{B}' = \{5, 7, 9, 16, 17, 31\}$
- $\mathcal{B}'' = \{3, 5, 7, 11, 31, 32\}$

Table: MAC Complexity for $\mathcal{B}$

MAC	Area (μm^2)	Area (%)[1]	Power (μW)	Power (%)[1]
modulo-5	14	4.6	5	4.1
modulo-7	28	9.3	14	11.6
modulo-31	70	23.3	37	30.6
modulo-32	33	11	16	13.2
modulo-33	156	51.8	49	40.4

Table: MAC Complexity for $\mathcal{B}'$

MAC	Area (μm^2)	Area (%)[1]	Power (μW)	Power (%)[1]
modulo-5	14	5.1	5	4.2
modulo-7	28	10.3	14	11.7
modulo-9	50	18.5	22	18.3
modulo-16	24	8.8	11	9.2
modulo-17	84	31.1	31	25.8
modulo-31	70	25.9	37	30.8

Table: MAC Complexity for $\mathcal{B}''$

MAC	Area (μm^2)	Area (%)[1]	Power (μW)	Power (%)[1]
modulo-3	10	4	4	3.8
modulo-5	14	5.6	5	4.7
modulo-7	28	11.3	14	13.3
modulo-11	93	37.5	29	27.7
modulo-31	70	28.2	37	35.2
modulo-32	33	13.3	16	15.2

Exploiting residue sparsity to reduce memory cost

- Commonly used compression schemes in sparse CNN architectures (CSC, CSR) are not suitable for the residue-sparse scenario
- Non-zero values occur at different indexes within the residue channels $\rightarrow$ multiple index vectors
- Utilize a variable-length encoding
- Compression ratio depends on the sparsity factors α_i.

$$G(d) = \begin{cases} 0, & \text{if } d = 0 \\ 1 d_{n-1} d_n \dots d_0, & \text{otherwise.} \end{cases} \tag{10}$$

The average size, $\widehat{n}$, of the encoded word assuming base $\mathcal{B}$ is given by

$$\begin{aligned} \widehat{n} &= \alpha_0 + (3+1)(1-\alpha_0) + \alpha_1 + (5+1)(1-\alpha_0) && (11) \\ &= 10 - 3\alpha_0 - 5\alpha_1. && (12) \end{aligned}$$

Using the sparsity factors $\alpha_0 = 0.8$ and $\alpha_1 = 0.14$, the average size of the encoded word is $\widehat{n} = 6.9$ bits, translating to a 13.75% compression ratio.

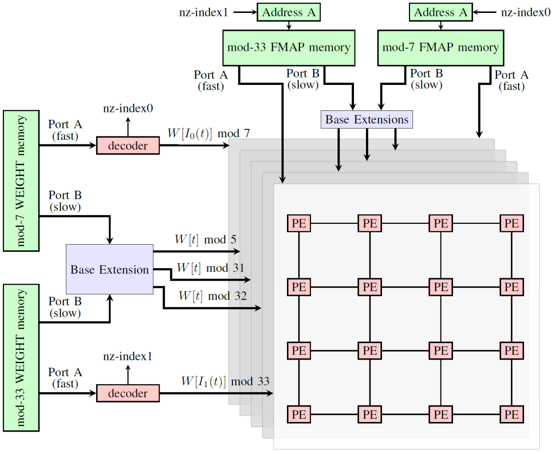
Exploiting residue sparsity to reduce power consumption

- Zero-skipping per moduli channel.
- The workloads of the different residue channels become unbalanced, since the regularization results in different sparsity levels
- Each channel may complete its computation at potentially different times
- Deactivate (power gating) a residue channel when it completes the processing
- Power savings depend on the achieved sparsity of each channel and its contribution to the total power consumption

Table: RNS base comparison

Base	$\mathcal{B}$	$\mathcal{B}'$	$\mathcal{B}''$
Power before regularization (μW)	121	120	105
Power after regularization (μW)	102.8	101.9	92.7
Savings (%)	15	15.1	11.7

Overall CNN architecture



- *n* (number of moduli) independent PE arrays of size $M \times M$
- A non-zero detector module reads a window of weight values (encoded with respect to $\mathcal{B}_{\text{weight}}$) and provides the next non-zero weight.
- The index of the next non-zero weight is used to select the corresponding input feature-map
- Base extension units are used to obtain the rest of the channels required for the convolution

Memory Organization

- Different sparsity levels $\rightarrow$ different processing rates for each channel
- Each channel requires a weight residue value from a different index of the weight vector at each timestep w_k^t
- $w_k^t = W[I_k(t)] \bmod m_k$, where W is the weight vector, $I_k(t)$ denotes the index of the weight required by the k-th channel $(0 \leq k < N)$ at time t.
- Base extension adds N_{be} channel to $\mathcal{B}_{weight}$ to obtain $\mathcal{B}$ $(N = N_w + N_{be})$.
- $I_k(t)$ are different for all the first N_w channels, while $I_k(t) = t$ for $k \geq N_w$
- Need to decouple the access to the weights of each residue channel $\rightarrow$ separate memory banks for each channel.
- At each timestep t, two weights from each channel are needed: one with an index of $I_k(t) \geq t$ and one with an index t required for the base extension $\rightarrow$ dual-port RAM macros.

Weight Decoder

- Performs the decompression of the weight values, according to the proposed variable-length compression scheme.
- For each channel in $\mathcal{B}_{weight}$ the weight decoder consists of a weight bit buffer, implemented with a programmable barrel shifter and a leading-one detector.
- The encoded weights are read from the corresponding weight memory bank and stored to the weight-bit buffer.
- The leading-one detector calculates the index of the next '1', corresponding to the next non-zero weight value, which determines the number of shift positions of the buffer.

Table: MAC and Decoder Complexity

Unit	Area (μm^2)	Power (μW)
Decoder $\mathcal{B}_{\text{weight}}$	85	45
MAC $\mathcal{B}$	301	121
Decoder $\mathcal{B}'_{\text{weight}}$	122	64
MAC $\mathcal{B}'$	270	120

- Decoder cost is small compared to MAC unit and amortized over a number if PEs
- For a 4×4 processing array the total decoder power consumption overhead is 2.3% and 3.3% for $\mathcal{B}_{weight}$ and $\mathcal{B}'_{weight}$

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Conclusions

- Modification of ANN training to induce increased residue-level sparsity in weights by regularization.
- $4\times$ to $6\times$ increase in residue sparsity in certain cases with minimal accuracy drop.
- Comparative evaluation of RNS bases in the context of the proposed method.
- Importance of the choice of RNS base for the exploitation of residue sparsity.
- Exploitation of residue-level sparsity to improve RNS-based hardware accelerators.
- Focus on decreasing energy requirements in hardware accelerators.
- Promising results for the final fully-connected layer of the VGG16 model.
- Potential for the proposed regularization technique to lead to new RNS architectures.
- Possibility of RNS becoming a candidate for hardware accelerators in edge devices.
- Future work: Optimize the method to scale efficiently with large models.

Thank you!